The condition $a_{m+2}=0$ can be written as

$$mV_0a_m+2p_2a_{m-2}=0$$

and, therefore,

$$V_0=\frac{1}{2}(m-1)p_2 \qquad \text{(A7)}$$

where, for small V_0 and p_2, we have used the expression $a_n=c_n$. Then, by using (A2), (A3), and (A7), (A6) can be written in the form

$$\Delta_{n+2}-\Delta_n=\frac{1}{2}(m+n-1)p_2, \qquad n \text{ even} \qquad \text{(A8)}$$

which yields

$$\Delta_n=\frac{1}{8}n(2m+n-4)p_2+\Delta_0. \qquad \text{(A9)}$$

By choosing Δ_0 so that $\Delta_m=0$, (A9) can be rewritten as

$$\Delta_n=-\frac{1}{8}(m-n)(3m+n-4)p_2. \qquad \text{(A10)}$$

References

[1] D. Marcuse, *Light Transmission Optics*. New York: Reinhold, 1972, pp. 263–285.

[2] S. E. Miller, "Light propagation in generalized lens-like media," *Bell Syst. Tech. J.*, vol. 44, pp. 2017–2064, Nov. 1965.

[3] E. A. I. Marcatili, "Off-axis wave-optics transmission in a lens-like medium with aberration," *Bell Syst. Tech. J.*, vol. 46, pp. 149–166, Jan. 1967.

[4] M. Hashimoto, "A perturbation method for the analysis of wave propagation in inhomogeneous dielectric waveguides with perturbed media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 559–566, Sept. 1976.

[5] T. Kitano, H. Matsumara, M. Furukawa, and I. Kitano, "Measurement of fourth-order aberration in a lens-like medium," *IEEE J. Quantum Electron.*, vol. QE-9, pp. 967–971, Oct. 1973.

The Screening Potential Theory of Excess Conduction Loss at Millimeter and Submillimeter Wavelengths

YEN-CHU WANG, MEMBER, IEEE

***Abstract*—On using the screening potential theory, the room-temperature excess conduction loss in copper waveguide is explained and calculated. The low-frequency and long-wavelength conductivity with spatial dispersion has been shown to give 30-percent more conduction loss in copper at submillimeter-wave frequency. Good agreement between experimental and theoretical results is obtained.**

RECENT measurements of the surface resistance of single-crystal copper by Tischer indicate that a room-temperature anomalous skin effect exists at millimeter-wave and upper microwave frequencies [1]–[3]. When extrinsic effects, i.e., surface roughness, waveguide size deviation, temperature, corrosion, work hardening, and oxygen absorption are taken into account and subsequently excluded, there is observed an anomalous skin effect which gives a 13.5-percent higher measured surface resistance than the classical theory can account for at 35 GHz and room temperature. It increases to 20 percent higher at 70 GHz. It is also reported for gold [5].

Manuscript received November 10, 1977; revised March 20, 1978. This work was supported by the NASA Goddard Space Flight Center.

The author is with the Department of Electrical Engineering, Howard University, Washington, DC 20059.

It is easily seen that the anomaly cannot be explained by either the Drude model or Pippard's anomalous skin effect. The anomaly can be instead attributed to the spatial dispersion of the conductivity $\sigma(q,\omega)$ due to the charge-density fluctuation induced screening potential [4]. The conductivity in the MKS units can be calculated from Harrison's dielectric function [4] (see Appendix) as given by

$$\sigma(q,\omega)=\sigma_r+i\sigma_i=-3i\omega\tau\sigma_0K/(qv\tau)^2 \qquad (1)$$

$$K=\frac{1-\dfrac{1-i\omega\tau}{2iqv\tau}\ln\left(\dfrac{1-i\omega\tau+iqv\tau}{1-i\omega\tau-iqv\tau}\right)}{1-\dfrac{1}{2iqv\tau}\ln\left(\dfrac{1-i\omega\tau+iqv\tau}{1-i\omega\tau-iqv\tau}\right)} \qquad (2)$$

where for copper $\tau=2.37\times10^{-14}$s, $v=v_F=1.58\times10^6$ m/s, $\sigma_0=e^2n_0\,\tau/\mathrm{m}=$ dc conductivity $=5.80\times10^7$ S/m; all field variables vary according to $\exp i\,(\boldsymbol{q}\cdot\boldsymbol{r}-\omega t)$. In deriving (1) and (2) it is assumed that the currents and fields will have the same dependence on position, which is reasonable because of the small mean-free path at low frequency and room temperature; consequently, electrons moving at all

0018-9480/78/1100-0858$00.75

angles with respect to the metal surface in the half-space will contribute to the conductivity within this semiclassical free-electron gas analysis.

On the other hand, the frequency considered is high, and the skin depth, although still larger than the mean-free path, is sufficiently close to it. The fields do vary appreciably over an electronic mean-free path, and the system response should be described by a conductivity which is not only frequency but also wavenumber dependent. The wavenumber q considered here is at least two orders of magnitude lower than the Fermi wavenumber. Only long-wavelength variations of potentials exist, and, therefore, the quantum-mechanical treatment is not needed. At long wavelengths, one does not concern oneself with the difference between conductivities for transverse and longitudinal fields in cubic materials. From (1) it is found that $\sigma(q,\omega)$ is proportional to $e^2/\epsilon_0 q^2$, the Fourier transform of the long-range coulomb energy, and also to K expressed in (2) which screens the potential and causes the conductivity to decrease and conduction losses to increase.

Since the transport problem of a semi-infinite metal involves complicated self-consistent response calculation, the wavenumber q is not known. It can be accurately determined, however, by fitting the experimental excess conduction loss data at the millimeter-wave frequencies to the screening model shown in (1) and (2). It is well known that the surface impedance is given by $Z=(\omega\mu_0/2\sigma)^{1/2}(1-i)$. For $\sigma(q,\omega)=\sigma_r+i\sigma_i$, it becomes

$$Z=\xi/\sigma_r\delta \tag{3}$$

$$\xi=\left[\cos\beta-\sin\beta-i(\cos\beta+\sin\beta)\right]/\left[1+(\sigma_i/\sigma_r)^2\right]^{1/4} \tag{4}$$

$$\delta=(2/\omega\mu_0\sigma_r)^{1/2} \tag{5}$$

$$2\beta=\tan^{-1}\sigma_i/\sigma_r. \tag{6}$$

The classical surface resistance R_0 is given by $R_0=(\omega\mu_0/2\sigma_0)^{1/2}$, and the screened semiclassical surface resistance R is given by the real part of Z. The excess losses are defined as $\Delta\alpha=R/R_0$, i.e.,

$$\Delta\alpha=\mathrm{Re}\,(\xi)(\sigma_0/\sigma_r)^{1/2}. \tag{7}$$

When σ_r and σ_i can be calculated from (1) and (2) by fitting the known experimental $\Delta\alpha$, the correct wavenumber, complex conductivity, surface resistance, and the theoretical excess loss can all be determined for each known experimental $\Delta\alpha$. Although it is known for only two frequencies, i.e., 35 and 70 GHz, extension toward the lower frequencies is obtained by extrapolation from the experimental data, and extension toward the higher frequencies is obtained by calculating $\Delta\alpha$ at $\omega\tau=1$ where anomalous skin effect becomes important. At $\omega\tau=1$, $\delta/l=0.68$ where l = mean-free path. Based on the ineffectiveness concept [6]–[8], $\sigma\simeq(\delta/l)\sigma_0$, and $\Delta\alpha=R/R_0\simeq(\sigma_0/\sigma)^{1/2}\simeq 1.21$. For all frequencies up to $\omega\tau=1$, it is found that q can be determined as follows.

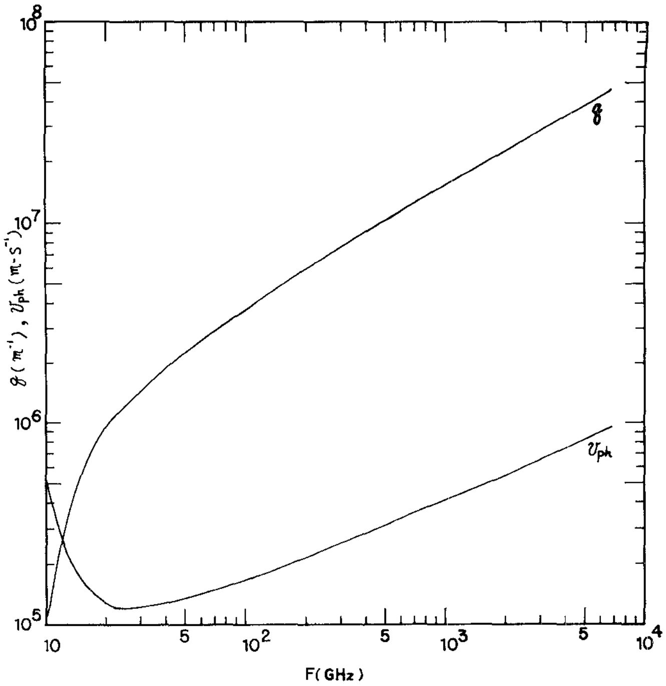

Fig. 1. Wavenumber q and phase veolcity v_{ph} versus frequency.

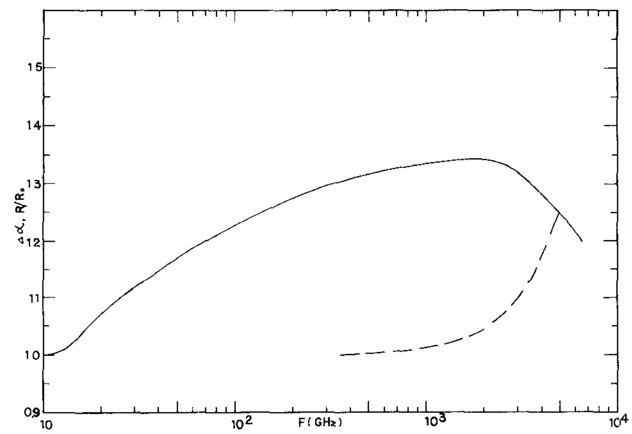

Fig. 2. Excess conduction loss $\Delta\alpha$ versus f as calculated from the screening theory. Dashed curve shows $\Delta\alpha$ based on Drude model.

For low frequency and long wavelengths,

$$\sigma(q,\omega)\simeq\sigma_0/\left[1+i(qv\tau)^2/3\omega\tau\right]. \tag{8}$$

Since the conduction loss does not involve σ_i, it can be considered as small. Let $x=\sigma_i/\sigma_r=-(qv\tau)^2/3\omega\tau\to 0$, then $\beta\to x/2$ from (6). $\Delta\alpha$ can be approximated to satisfy $x^2+x+1-\Delta\alpha=0$ from (7). Thus

$$q=\left\{\left[1+4(\Delta\alpha-1)\right]^{1/2}-1\right\}^{1/2}(3\omega\tau)^{1/2}/v\tau. \tag{9}$$

Substituting (9) into (1) gives σ_r, R, and the theoretical excess loss.

As shown in Fig. 1, the wavenumber q and the phase velocity $V_{ph}=\omega/q$ are calculated by fitting data as discussed above. Between 70 GHz and the highest frequency 6715.4 GHz where $\omega\tau=1$, a monotonic increasing function has been assumed in the q–f plot, which will be justified later. In Fig. 2, the excess conduction loss $\Delta\alpha$ is plotted against the frequency. Calculation is based on the values of q given in Fig. 1. $\Delta\alpha$ increases with frequency until $f \simeq 2000$ GHz is reached, beyond which $\Delta\alpha$ tends to decrease. As these frequencies become closer to the anomalous skin effect region and less electrons partake in conduction, $\Delta\alpha$ should tend to the anomalous value as shown. It is therefore reasonable to assume a monotonic increasing wavenumber in Fig. 1. The dashed curve represents the excess conduction loss calculated from the Drude model where $\sigma_r = \sigma_0/(1+\omega^2\tau^2)$. In Fig. 3, R, σ_r, and δ are calculated from the conductivity with spatial dispersion due to the screening potential effect. R_0 and δ_0 are the classical surface resistance and skin depth, respectively.

The total potential $V(q,\omega)$ normalized to the applied potential is given by $V(q,\omega) = \epsilon^{-1}(q,\omega)$, $\epsilon(q,\omega) = 1 - \sigma(q,\omega)/i\omega\epsilon_0$. On using (8) and $\sigma_0 \gg \omega\epsilon_0$, $V(r,\omega) = C_1\delta(r)/r^2 + C_2 \exp(-\kappa_{FT} r)/r$, where C_1 and C_2 are constants, $\kappa_{FT}^2 = (3/2)n_0 e^2/\epsilon_0 E_F$. The potential is screened exponentially and characterized by the Fermi–Thomas screening parameter κ_{FT}. When $(qv\tau)^2/3\omega\tau \ll 1$ and $\sigma_0 \gg \omega\epsilon_0$, the potential is found to be proportional to $\cos(\kappa r)\exp(-\kappa r)/r$ and is oscillatory at small r where $\kappa = (3/2)\omega\tau^{1/2}/l$, l = mean-free path.

Although calculation of excess conduction loss in this work takes copper as an example, it should be equally applicable to other normal metals.

Appendix

It is well known that the electric field tangential to a semi-infinite conductor surface does not induce surface charges; the field normal to the surface does. The induced fluctuation in electron density inside the metal will cause an additional electrostatic potential seen by the electron. It is called the screen potential. Thus, within the self-consistent field approximation, the total potential includes both the applied and screening terms of the form $V_t \exp i(\mathbf{q}\cdot\mathbf{r} - \omega t)$. (Inside, the metal speed of the wave motion of density is much less than that of light; a quasi-static approach will suffice.) Solving for the total potential from Poisson's equation and the fluctuation in the electron density from Boltzmann's equation self-consistently, Harrison [4] obtains the dielectric function $\epsilon(q,\omega)$ which is the ratio of the applied to the total potential

$$\epsilon(q,\omega) = 1 + \frac{e^2}{\epsilon_0 q^2} n(\xi) K \tag{A1}$$

where K is shown in (2). $n(\xi) = 8\pi p^2/h^3 v$ = density of states at the Fermi energy where $p = \hbar(3\pi^2 n_0)^{1/3} = mv$, and n_0 = electron density. We get

$$n(\xi) = 3n_0/mv^2. \tag{A2}$$

Since

$$\sigma(q,\omega) = i\omega\epsilon_0[1 - \epsilon(q,\omega)] \tag{A3}$$

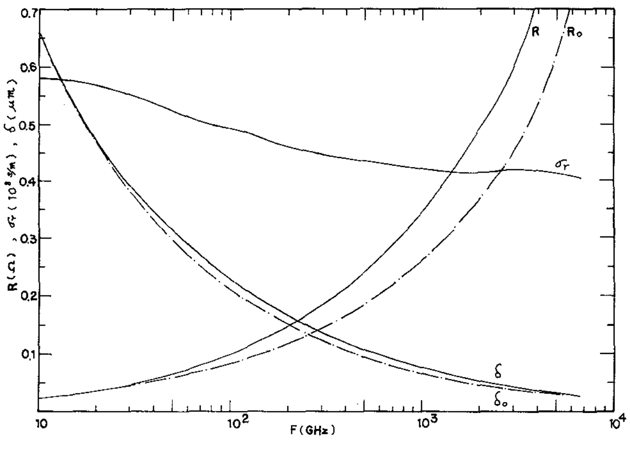

Fig. 3. Surface resistance R, real part of conductivity σ_r, and skin depth δ based on the screening theory versus f. Comparisons with the classical R_0 and δ_0 are shown in dashed curves.

and $\sigma_0 = e^2 n_0 \tau/m$, (A1) and (A2) can be used to give (1) using (A3).

The Drude model [4], [7], and [8] can be obtained from (1) and (2) considering the long-wavelength limit, $q \to 0$, i.e., uniform field. Since

$$\ln[(1+x)/(1-x)] \cong 2x + 2x^3/3 \tag{A4}$$

$$x = iqv\tau/(1 - i\omega\tau) \tag{A5}$$

the factor $K \to (qv\tau)^2/-3i\omega\tau(1-i\omega\tau)$ and the conductivity of the Drude model [4] becomes

$$\sigma = \sigma_0/(1 - i\omega\tau). \tag{A6}$$

The real part of σ gives rise to loss and is given by

$$\sigma_r = \sigma_0/(1+\omega^2\tau^2) \tag{A7}$$

which decreases with increasing frequencies. However, it can account for neither the 20-percent higher excess loss at 70 GHz nor the 13.5-percent higher excess loss at 35 GHz because $(\omega\tau)^2 = 1.1 \times 10^{-4}$ at 70 GHz and $(\omega\tau)^2 = 2.7 \times 10^{-5}$ at 35 GHz, which are negligibly small. Pippard's anomalous skin effect [6]–[8] sets in only at sufficiently low temperature and/or high frequencies. For Cu 300 K it should become important at frequencies higher than 1500 GHz [8]. It has no significance at 35 and 70 GHz.

Acknowledgment

The author is grateful to the Solid State Electronics Technology Development Program, Howard University, Washington, DC, and the Science Center, Rockwell International, for research support.

References

[1] F. J. Tischer, "Measurement of the surface resistance of single-crystal copper in the millimeter-wave region at room temperature," *Appl. Phys. Lett.*, vol. S, pp. 285–286, 1974.

[2] ——, "Excess conduction losses at millimeter wavelengths," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 853–858, Nov. 1976.

[3] ——, "Experimental attenuation of rectangular waveguides at millimeter-wavelengths," *IEEE Int. Microwave Symp. Digest* (San Diego, CA), June 1977, pp. 492–494.

[4] W. A. Harrison, *Solid State Theory*. New York: McGraw-Hill, 1970, pp. 280–290.
[5] R. J. Batt, G. D. Jones, and D. J. Harris, "The Measurement of the surface resistivity of evaporated gold at 890 GHz," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 488–491, June 1977.
[6] R. Kubo and T. Nagamiya, Eds., *Solid State Physics*. New York: McGraw-Hill, 1969, pp. 245–249.
[7] A. A. Abrikosov, *Introduction to The Theory of Normal Metals, Solid State Physics*, H. Ehrenreich, F. Seitz, and D. Turnball, Eds., Suppl. 12. New York: Academic, 1972, ch. 5.
[8] F. Wooten, *Optical Properties of Solids*. New York: Academic, 1972, ch. 4.

The Traveling-Wave IMPATT Mode

MICHAEL FRANZ AND JAMES B. BEYER, MEMBER, IEEE

***Abstract*—The small-signal analysis of a distributed IMPATT diode ifdicates the existence of a traveling-wave mode. The severe power-frequency limitation as well as the associated low impedance level of the discrete diode appear avoidable. No external resonant circuitry is needed.**

It is shown that the TEM parallel-plate waveguide mode of the junction is modified by the injection of electrons at the p^+-n junction (or Schottky contact). The transverse electric field takes on a traveling-wave nature in the transverse direction tracking the injected electrons, and a small longitudinal electric field will also be present.

In previous papers on IMPATT traveling-wave structures, the IMPATT effect was lumped into an effective complex permittivity in a composite layer model or into an effective shunt admittance in a transmission line model. The current work attempts to incorporate the IMPATT mechanism into the wave model and considers the actual carrier field interaction.

The small-signal analysis yields an analytic field solution and a characteristic equation for the complex propagation constant. Solutions are found and documented for various frequencies and bias current densities. For the particular structure considered, at 12 GHz with a bias current density of 1000 A/cm² a gain of 72 dB/cm was found.

I. INTRODUCTION

THE QUANTITATIVE model of traveling-wave behavior in an IMPATT diode is based on the idealized structure of Fig. 1. The diode shown is essentially an IMPATT diode elongated in the z direction, wherein the p^+-n^+-n sandwich of the diode is contacted by two metallic layers.

Due to the distributed nature of the device, the power-frequency limitation associated with a lumped diode will not apply with the additional advantage that no external resonant circuitry is needed, which points to the possibility of direct integration. The application of the diode as an amplifier, modulator (phase shifter), or as an oscillator would depend on the coupling and matching scheme used.

To obtain a qualitative understanding of the traveling-wave behavior, it is assumed that the diode of Fig. 1 is reversed biased and that a TEM parallel-plate waveguide mode exists in the depletion layer. Neglecting fringing effects, the electric ac field will be given by

$$E_x = E_0 e^{\gamma z} \tag{1}$$

with γ as the wavenumber.

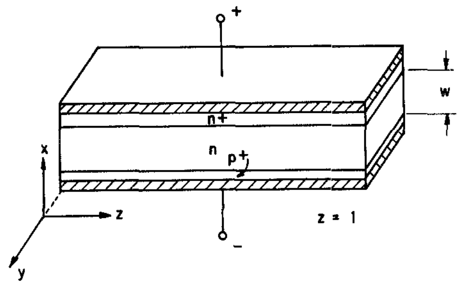

Fig. 1. Distributed IMPATT diode.

An instantaneous picture of E_x is shown in Fig. 2(a). In IMPATT operation, the avalanche carrier built up at the p^+-n junction will reach its maximum after the completion of the negative half-cycle. At the transit time frequency

$$f = \frac{v_s}{2w} \tag{2}$$

the electrons cross the depletion region with scattering limited velocity during the positive half-cycle and are collected by the n^+ substrate. The front of maximum electron density is symbolized in Fig. 2(b) for a wave traveling to the right. The traversing electrons constitute a current opposing the ac electric field and contribute RF power to the wave.

Thus the principles of power transfer in the distributed diode from a dc source to an RF signal are the same as those in the lumped diode, only here the IMPATT process propagates with the wave. These principles can be summarized as follows.

1) In the depletion region of the diode, power is added to an existing RF electric field by forcing charge carriers

Manuscript received December 9, 1977; revised March 23, 1978.
The authors are with the Department of Electrical and Computer Engineering, University of Wisconsin-Madison, Madison, WI 53706.

0018-9480/78/1100-0861$00.75 ©1978 IEEE